

Direct detection of supersymmetric dark matter- Theoretical rates for transitions to excited states.

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The recent WMAP data have confirmed that exotic dark matter together with the vacuum energy (cosmological constant) dominate in the flat Universe. Supersymmetry provides a natural dark matter candidate, the lightest supersymmetric particle (LSP). Thus the direct dark matter detection is central to particle physics and cosmology. Most of the research on this issue has hitherto focused on the detection of the recoiling nucleus. In this paper we study transitions to the excited states, focusing on the first excited state at 50 keV of Iodine A=127. We find that the transition rate to this excited state is ≤ 10 percent of the transition to the ground state. So, in principle, the extra signature of the gamma ray following its de-excitation can be exploited experimentally.

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INTRODUCTION

The combined MAXIMA-1 [1], BOOMERANG [2], DASI [3], COBE/DMR Cosmic Microwave Background (CMB) observations [4], the recent WMAP data [6] and SDSS [7] imply that the Universe is flat [5] and that most of the matter in the Universe is dark, i.e. exotic.

$$\Omega_b = 0.044 \pm 0.04, \Omega_m = 0.27 \pm 0.04, \Omega_\Lambda = 0.69 \pm 0.08$$

for baryonic matter, cold dark matter and dark energy respectively. An analysis of a combination of SDSS and WMAP data yields [7] $\Omega_m \approx 0.30 \pm 0.04(1\sigma)$. Crudely speaking

$$\Omega_b \approx 0.05, \Omega_{CDM} \approx 0.30, \Omega_\Lambda \approx 0.65$$

Since the non exotic component cannot exceed 40% of the CDM [8], there is room for the exotic WIMP's (Weakly Interacting Massive Particles). In fact the DAMA experiment [9] has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal [10], although these data are not consistent with other recent experiments, see e.g. EDELWEISS [11] and CDMS [12].

Supersymmetry naturally provides candidates for the dark matter constituents [13],[14]-[17]. In the most favored scenario of supersymmetry the LSP can be simply described as a Majorana fermion, a linear combination of the neutral components of the gauginos and higgsinos [13],[14]-[21]. In most calculations the neutralino is assumed to be primarily a gaugino, usually a bino. Models which predict a substantial fraction of higgsino lead to a relatively large spin induced cross section due to the Z-exchange. Such models tend to violate the LSP relic abundance constraint and are not favored. Some claims have recently been made, however, to the effect that the WMAP relic abundance constraint can be satisfied in the hyperbolic branch of the allowed SUSY parameter space, even though the neutralino is then primarily a higgsino [22]. We will not elaborate further on this point, but we will take the optimistic view that the detection rates due to the spin may be large enough to be exploited by the experiments, see, e.g., [22] [23], [24], [25]. Such a view is further encouraged by the fact that, unlike the scalar interaction, the axial current allows one to populate excited nuclear states, provided that their energies are sufficiently low so that they are accessible by the low energy LSP, a prospect proposed long time ago by Ejiri and collaborators [26]. As a matter of fact the average kinetic energy of the LSP is:

$$\langle T \rangle \approx 40 \text{ keV} \frac{m_\chi}{100 \text{ GeV}} \quad (1)$$

So for sufficiently heavy LSP the average energy may exceed the excitation energy, e.g. of about 50 keV for the excited state of ^{127}I . In other words one can explore the high velocity window, up to the escape velocity of $v_{esc} \approx 570 \text{ km/s}$. From a Nuclear Physics point of view this transition is not expected to be suppressed, since it is of the type $(5/2)^+ \rightarrow (7/2)^+$, i.e. Gamow-Teller like.

THE LSP-NUCLEUS CROSS SECTION

The LSP-nucleus differential cross section with respect to the energy transfer Q for a given LSP velocity v due to the spin can be cast in the form

$$d\sigma(u, v) = \frac{du}{2(\mu_r bv)^2} \bar{\Sigma}_{spin} F_{11}(u) \quad (2)$$

where F_{11} is the spin response of the isovector channel [27]. We have used a dimensionless variable u , proportional to Q , which has been found convenient for handling the nuclear form factor [31] $F(u)$, namely:

$$u = \frac{Q}{Q_0} \quad , \quad Q_0 = \frac{1}{Am_N b^2} \approx 40 \times A^{-4/3} \text{ MeV}. \quad (3)$$

μ_r is the reduced LSP-nucleus mass and b is the (harmonic oscillator) nuclear size parameter.

The quantity $\bar{\Sigma}_{spin}$ is given by:

$$\bar{\Sigma}_{spin} = \left(\frac{\mu_r}{\mu_r(p)} \right)^2 \sigma_{p, \chi^0}^{spin} \zeta_{spin}, \zeta_{spin} = \frac{1}{3(1 + \frac{f_A^0}{f_A^1})^2} S(u) \quad (4)$$

$\mu_r(p) \approx m_p$ is the LSP-nucleon reduced mass and $\sigma_{p, \chi^0}^{spin}$ is the proton cross-section associated with the spin and $S(u)$ [27] is given by:

$$S(u) = [(\frac{f_A^0}{f_A^1} \Omega_0(0))^2 \frac{F_{00}(u)}{F_{11}(u)} + 2 \frac{f_A^0}{f_A^1} \Omega_0(0) \Omega_1(0) \frac{F_{01}(u)}{F_{11}(u)} + \Omega_1(0)^2] \quad (5)$$

The overall normalization of the axial current components is not important in the present discussion. We mention, however, that they have been normalized so that $\sigma_{p, \chi^0}^{spin} = 3(f_A^0 + F_A^1)^2 \sigma_0$, $\sigma_{n, \chi^0}^{spin} = 3(f_A^0 - F_A^1)^2 \sigma_0$, $\sigma_0 = \frac{1}{2\pi} (G_F m_p)^2$, for the proton and neutron spin cross sections respectively.

Notice that $S(u)$, being dependent on ratios, is expected to be less dependent on the energy transfer and the scale of supersymmetry. As a matter of fact it has been found [27] that the spin response functions F_{ij} , the "spin form factors" normalized to unity at $u = 0$, have similar dependence on the energy transfer. In other words $S(u)$ is essentially independent of u .

A number of nuclear spin matrix elements for a variety of targets have become available [27]-[36]. Some static spin matrix elements [27], [29], [31] for some nuclei of interest are given in table I. The spin matrix elements appearing in the table are defined as:

$$\Omega_0 = \Sigma_p + \Sigma_n \quad , \quad \Omega_1 = \Sigma_p - \Sigma_n \quad , \quad \Sigma_k = \frac{1}{\sqrt{2J_i + 1}} < J_f || 2 \mathbf{S}_k || J_i > \quad , \quad k = p, n \quad , \quad (6)$$

where by double bar we indicate the reduced matrix element as defined in standard textbook as e.g the one by Edmonds. The $< S_k >$, $k = p, n$, are defined in the literature, e.g. Ressel and Dean [29].

In order to proceed, in addition to the static spin matrix elements and the spin response functions F_{00}, F_{01}, F_{11} ("spin form factors"), normalized to unity at zero momentum transfer, one must know the isoscalar and the isovector axial currents. In the allowed supersymmetric parameter space one obtains the isoscalar and the isovector axial current components at the quark level. Going from the quark to the nucleon level is straightforward for the isovector current, but it not trivial in the case of the isoscalar current, since the naive quark model fails badly. Most of the proton spin is not due to the quark spins (proton spin crisis-EMC effect). Thus one finds:

$$f_A^0(q) \rightarrow f_A^0 = g_A^0 \quad f_A^0(q) \quad , \quad f_A^1(q) \rightarrow f_A^1 = g_A^0 \quad f_A^1(q) \quad , \quad g_A^0 \approx 0.1 \quad , \quad g_A^1 = 1.23 \quad (7)$$

It thus appears that, barring very special circumstances, whereby the isoscalar contribution at the quark level, $f_A^0(q)$, is much larger than the isovector, $f_A^1(q)$, the isoscalar contribution can be neglected.

THE STRUCTURE OF ^{127}I

This nucleus has a ground state $5/2^+$ and a first excited state a $7/2^+$ at 57.6 keV . As it has already been mentioned it is a popular target for dark matter detection. As a result the structure of its ground state has been studied

theoretically by a lot of groups. Among them we mention again the work of Ressel and Dean [29], the work of Engel, Pittel and Vogel [35], Engel and Vogel [32], Iachello, Krauss and Maiano [33], Nikolaev and Klapdor-Kleingrothaus [34] and more recently by Suhonen and collaborators [36]. In all these calculations it appears that the spin matrix element is dominated by its proton component, which in our notation implies that the isoscalar and the isovector components are the same. In these calculations there appears to be a spread in the spin matrix elements ranging from 0.07 up to 0.354, in the notation of Ressel and Dean [29]. This, of course, implies discrepancies of about a factor of 25 in the event rates.

In the present work we are primarily interested in the spin matrix element connecting the ground state to the first excited state. As we have mentioned, however, it is advantageous to compute the branching ratio. In addition to factoring out most of the uncertainties connected with the SUSY parameters and the structure of the nucleon, we expect the ratio of the two spin matrix elements to be more reliable than their absolute values. Thus this ratio is going to be the most important factor in designing the experiments to detect the rate to the excited state.

In order to have for $Z = 53$ single particle orbits suitable for the above states, close to the Fermi surface [37], namely $[402, 5/2]$ and $[404, 7/2]$ proton orbits, one should consider oblate deformation [38]. The value of the deformation, however, is not very well defined. To this end one should explore the spectrum of this system and combine information obtained from it with data on the neighboring systems Te and Xe . One notices that on the above band heads $\Delta J = 2$ rotation aligned coupling bands are built [39]. From the spectra one may also conclude that the $7/2^+$ data most likely correspond to a deformed band. The $5/2$ data correspond to a highly perturbed band or to a spherical vibrator. Other people have considered triaxial shapes, see ref. 9 of Shroy *et al* [39]. So we will take it as an educated guess that a description in terms of a rotor plus a particle with relatively large oblate deformation provides a reasonable description of this nucleus.

Given the above framework one may eventually have to do a self-consistent calculation, which will yield both the single particle states and the equilibrium deformation. At this exploratory stage, however, we were merely interested in testing the rather crude collective model contentions discussed above. To that effect we found it appropriate to take as simple, and easy to read, as possible the single particle substrate of our description. Thus to estimate the spin matrix elements, in the present simple spectroscopic study, we have proceeded in the following oversimplified fashion:

- Make a big leap in history to Nilsson's thesis [37].
- Compute the spin proton matrix elements involving the above bare single particle states for three choices of oblate deformations.
- include the effect of the Coriolis coupling for the $7/2^+$.

The relevant deformation parameter η is defined [37] as

$$\eta \approx \frac{\delta (1 - (4/3)\delta^2)}{\chi} \quad (8)$$

where δ is the usual Nilsson deformation parameter and $\chi \approx 0.05$. The values of η considered are $-6, -4, -2$. To first order the corresponding δ values are: $-0.3, -0.2, -0.1$. The corresponding states in the standard notation $N\ell\Lambda\Sigma$ are given by:

$$|5/2 \rangle = \frac{a|442+ \rangle + b|422+ \rangle + c|443- \rangle}{\sqrt{a^2 + b^2 + c^2}} \quad (9)$$

$$|7/2 \rangle = \frac{d|443+ \rangle + e|444- \rangle}{\sqrt{d^2 + e^2}} \quad (10)$$

The numerical values for these coefficients in the above order of the η 's are:

$$\eta = -6 \rightarrow (a, b, c, d, e) = (1.000, 0.552, -0.809, 1.000, -1.192)$$

$$\eta = -4 \rightarrow (a, b, c, d, e) = (1.000, 0.558, -1.081, 1.000, -1.662)$$

$$\eta = -2 \rightarrow (a, b, c, d, e) = (1.000, 0.517, -1.451, 1.000, -2.219)$$

Thus in the standard angular momentum notation $|J, M\rangle$ one obtains:

$$\langle 5/2, 5/2 | s_z | 5/2, 5/2 \rangle = 0.166, 0.033, -0.124 \quad (11)$$

$$\langle 7/2, 7/2 | s_z | 7/2, 7/2 \rangle = -0.087, -0.234, -0.331 \quad (12)$$

$$\langle 5/2, 5/2 | s_+ | (7/2, 5/2) \rangle = 0.171, -0.120, -0.073 \quad (13)$$

The array of numbers in the above three equations corresponds to $\eta = -6, -4, -2$ respectively.

One may also have to consider the effect of the Coriolis coupling, see e.g. [40], between the $I = 7/2$ and the $K = 5/2, 7/2$ bare single particle states with the Hamiltonian matrix as follows:

$$M_{ii} = E_i^{bare} + \frac{\hbar^2}{2J} [I(I+1) - 2K^2] \quad (14)$$

for the diagonal term. For the off-diagonal term one has:

$$V = \frac{\hbar^2}{J} \left[\frac{I(I+1) - K_<(K_<+1)}{2} \right]^{1/2} \langle K_> | j_+ | K_< \rangle \quad (15)$$

where j_+ is given in units of \hbar . One, of course, has to adopt a reasonable value for the moment of inertia J . For such a choice and $\eta = -6$ one finds:

$$M_{11} = 387, M_{22} = 145, V = 207 \quad (16)$$

(in KeV). The resulting components of the eigenfunctions are:

0.50, 0.87 and $-0.87, 0.50$ for the $5/2$ and $7/2$ respectively.

With the above wave functions we find the ground state spin matrix elements:

$$\langle S_p \rangle = 0.166, \quad \langle S_n \rangle = 0$$

, which are in good agreement with Iachello *et al* [33] and Nikolaev and Klapdor-Kleingrothaus [34], but substantially smaller than Ressel and Dean [29]. In our notation this leads to:

$$\Omega_0^2 = \Omega_1^2 = \Omega_0 \Omega_1 = 0.164 \quad (17)$$

This value is also smaller than the more recent result [36]. For the transition to the excited state we have obtained:

$$\Omega_0^2 = \Omega_1^2 = \Omega_0 \Omega_1 = 0.312 \quad (18)$$

Thus the spin matrix element, left to itself, seems to yield an enhancement of about a factor of two for the transition to the excited state. In our simple model this result is independent of the ratio of the isoscalar to isovector amplitude.

EXPRESSIONS FOR THE RATES

In this section we will briefly outline the expressions yielding the event rates, both modulated and unmodulated, (for more details see our earlier work [25] and references therein). The differential non directional rate can be written as

$$dR = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} d\sigma(u, v) |v| \quad (19)$$

where $d\sigma(u, v)$ was given above, $\rho(0)$ is the LSP density in our vicinity, m_χ the LSP mass, m is the mass of the target and A is the nuclear mass number.

For a given velocity distribution $f(\mathbf{v}')$, with respect to the center of the galaxy, one can find the velocity distribution in the lab frame $f(\mathbf{v}, \mathbf{v}_E)$ by writing

$$\mathbf{v} = \mathbf{v} + \mathbf{v}_E, \quad \mathbf{v}_E = \mathbf{v}_0 + \mathbf{v}_1$$

where $\mathbf{v}_0 = 229 \text{ km/s}$ is the sun's velocity (around the center of the galaxy) and \mathbf{v}_1 the Earth's velocity (around the sun). The velocity of the earth is given by

$$\mathbf{v}_E = \mathbf{v}_0 \hat{z} + \mathbf{v}_1 (\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z}) \quad (20)$$

In the above formula \hat{z} is in the direction of the sun's motion, \hat{x} is in the radial direction out of the galaxy, \hat{y} is perpendicular in the plane of the galaxy ($\hat{y} = \hat{z} \times \hat{x}$) and $\gamma \approx \pi/6$ is the inclination of the axis of the ecliptic with respect to the plane of the galaxy. α is the phase of the Earth in its motion around the sun ($\alpha = 0$ around June 2nd). The time dependence of the event rate, modulation effect, will be present for transitions to the excited states as well.

Folding with the velocity distribution we get:

$$\left\langle \frac{dR_{undir}}{du} \right\rangle = \left\langle \frac{dR}{du} \right\rangle = \frac{\rho(0)}{m_\chi} \frac{m}{Am_N} \sqrt{\langle v^2 \rangle} \left\langle \frac{d\Sigma}{du} \right\rangle \quad (21)$$

where

$$\left\langle \frac{d\Sigma}{du} \right\rangle = \int \frac{|v|}{\sqrt{\langle v^2 \rangle}} f(\mathbf{v}, \mathbf{v}_E) \frac{d\sigma(u, v)}{du} d^3\mathbf{v} \quad (22)$$

In our calculation we have used the standard M-B distribution:

$$f(\mathbf{v}') = \frac{1}{(\pi v_0^2)^{3/2}} \text{Exp} \left[-\frac{(\mathbf{v}')^2}{v_0^2} \right] \quad (23)$$

In what follows we will express the LSP velocity in units of v_0 introducing the dimensionless quantity $y = v/v_0$. Incorporating the relevant kinematics and integrating the above expression, Eq. (22), from u_{min} to u_{max} we obtain the total rates as follows:

$$R_{gs} = \int_{u_{min}}^{u_{max}} \left\langle \frac{dR_{gs}}{du} \right\rangle du \quad (24)$$

with $u_{min} = Q_{min}/Q_0$, Q_{min} imposed by the detector energy cutoff and $u_{max} = (y_{esc}/a)^2$, with $a = [\mu_r b \sqrt{2}]^{-1}$, is imposed by the escape velocity ($y_{esc} = 2.84$).

$$R_{exc} = \int_{u_{exc}}^{u_{max}} \left\langle \frac{dR_{exc}}{du} \left(1 - \frac{u_{exc}^2}{u^2} \right) \right\rangle du \quad (25)$$

where $u_{exc} = \frac{\mu_r E_x}{Am_N Q_0}$ and E_x is the excitation anergy of the final nucleus. The upper limit is now given by $u_{max} = (y/a)^2 - (E_x/Q_0)$

BRANCHING RATIOS

In the present calculation, to spare the reader with details about the SUSY allowed parameter space, we are not going to discuss the absolute value of the event rates. We will, instead, estimate the ratio of the rate to the excited state divided by that to the ground state (branching ratio of the rates) as a function of the LSP mass. This is of interest to the experimentalists. It can be cast in the form:

$$BRR = \frac{S_{exc}(0)}{S_{gs}(0)} \frac{\Psi_{exc}(u_{exc}, u_{umax}) [1 + h_{exc}(u_{exc}, u_{umax}) \cos \alpha]}{\Psi_{gs}(u_{min}) [1 + h(u_{min}) \cos \alpha]} \quad (26)$$

In the above expression h and h_{exc} are the modulation amplitudes for the ground state and excited state respectively. They correspond to the ratio of the amplitude associated with the motion of the Earth divided by the one which is independent of the motion of the Earth. The parameter α is the phase of the Earth in its motion around the Sun ($\alpha = 0$ on June 3rd). The modulation amplitudes can only be obtained numerically and are not going to be discussed in detail here (for more information see our earlier work [25]).

$S_{gs}(0)$ and $S_{exc}(0)$ are the static spin matrix elements evaluated via Eq. 5. This ratio of the static spin matrix elements is essentially independent of supersymmetry, if the isoscalar contribution is neglected due to the EMC effect. The functions Ψ are given as follows :

$$\Psi_{gs}(u_{min}) = \int_{u_{min}}^{(y/a)^2} \frac{S_{gs}(u)}{S_{gs}(0)} F_{11}^{gs}(u) [\psi(a\sqrt{u}) - \psi(y_{esc})] du \quad (27)$$

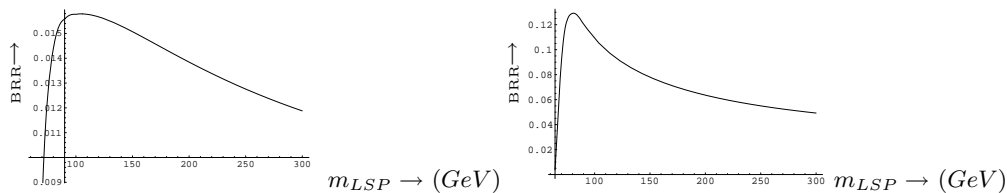


FIG. 1: The ratio of the rate to the excited state divided by that of the ground state as a function of the LSP mass (in GeV) for ^{127}I . We assumed that the static spin matrix element of the transition from the ground to the excited state is a factor of 1.9 larger than that involving the ground state, but the spin response functions $F_{11}(u)$ are the same. On the left we show the results for $Q_{min} = 0$ and on the right for $Q_{min} = 10 \text{ KeV}$.

$$\Psi_{exc}(u_{exc}, u_{max}) = \int_{u_{exc}}^{u_{max}} \frac{S_{exc}(u)}{S_{exc}(0)} F_{11}^{exc}(u) \left(1 - \frac{u_{exc}^2}{u^2}\right) [\psi(a\sqrt{u}(1 + u_{exc}/u)) - \psi(y_{esc})] du \quad (28)$$

The functions ψ were evaluated numerically, with the correct kinematics imposed on the neutralino velocity in the laboratory, which is obtained by translating the simple relation $v' \leq v_{esc}$, which holds with respect to the galactic center. To a very good approximation, however, they can be given by:

$$\psi(z) = \frac{1}{2} [\text{Erf}(z+1) - \text{Erf}(z-1)] \quad (29)$$

and with $\text{Erf}(x)$ the well known error function:

$$\text{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

As we have mentioned the dependence of $S(u)$ on the energy transfer is extremely mild and it can be neglected.

CONCLUSIONS

In the present paper we have calculated the branching ratio BBR for the neutralino - ^{127}I scattering event rate to the first excited state. We found that the ratio of the static spin matrix element involving the excited state is 1.9 times larger than that of the ground state. Assuming that the spin response functions $F_{11}(u)$ are identical, we obtained the ratio BRR (branching ratio of the rate to the excited divide by that to the ground state), which is exhibited in Fig. 1. In computing these branching ratios we did not include the quenching factors affecting the nuclear recoils, i.e. the detected event rates for transitions to the ground state. If such factors are included, the branching ratios will increase.

The difference in the branching ratios of Fig. 1 can be understood as follows: Since the transitions the excited state will be detected not by nuclear recoil, but via the EM decay of the final state, they do not suffer from the threshold energy cut off. This cut off suppresses only the transitions to the ground state, which are detected through nuclear recoils. Anyway the branching ratio is affected. Thus this ratio is very small in the, at present, unattainable experimentally case of $Q_{min} = 0$. As we have mentioned, however, even in this case, the branching ratio will increase, if the quenching factor is taken into account. The branching ratio is small even in the case of $Q_{min} = 10 \text{ keV}$ for LSP masses less than 50 GeV. This is not unexpected, since such a light neutralino cannot provide the energy needed to excite the nucleus. For $Q_{min} = 10$ this branching ratio attains a maximum value, around 10%, for LSP masses around 100 GeV and gets values around 6% for larger masses. For larger Q_{min} the branching ratio will further increase.

The smallness of the branching ratio, $\leq 10\%$, should not discourage the experiments, since the transition to the excited state offers experimental advantages. This is especially true, if the traditional experiments cannot reach $Q_{min} \leq 10 \text{ keV}$.

Finally from Figs 2 and 3 we notice that, especially for $m_\chi \gtrsim 100 \text{ GeV}$, the modulation amplitude for transitions to the excited state is much bigger compared to that of the elastic scattering. We hope that this additional signal can perhaps be exploited by the experimentalists to discriminate against background γ rays, in a fashion analogous to the ongoing measurements of nuclear recoils.

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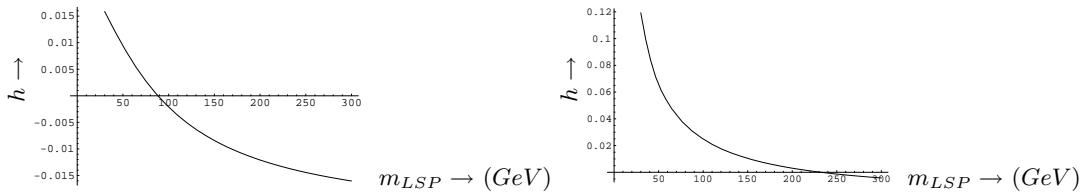


FIG. 2: The the same as in Fig. 1 for the modulation amplitude h for the transition to the ground state. For the definitions see text.

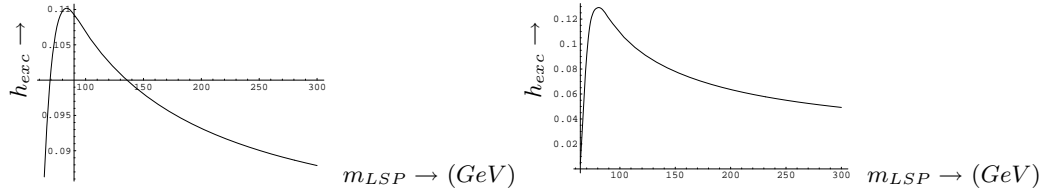


FIG. 3: The the same as in Fig. 2 for the modulation amplitude h_{exc} for the transition to the excited state.

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TABLE I: The static spin matrix elements for various nuclei. For light nuclei the calculations are from Divari et al [27] . For ^{127}I the results are from Ressel and Dean [29] (*) and Homlund *et al* [36](**). For ^{207}Pb they were obtained previously [18], [31].

	^{19}F	^{29}Si	^{23}Na	$^{127}\text{I}^*$	$^{127}\text{I}^{**}$	$^{207}\text{Pb}^+$
$[\Omega_0(0)]^2$	2.610	0.207	0.477	3.293	1.488	0.305
$[\Omega_1(0)]^2$	2.807	0.219	0.346	1.220	1.513	0.231
$\Omega_0(0)\Omega_1(0)$	2.707	-0.213	0.406	2.008	1.501	-0.266
μ_{th}	2.91	-0.50	2.22			
μ_{exp}	2.62	-0.56	2.22			
$\frac{\mu_{th}(spin)}{\mu_{exp}}$	0.91	0.99	0.57			

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